11/14 Recitation

1. Let

$$A = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{5} & 0 & 0 & 2/\sqrt{5} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -2/\sqrt{5} & 0 & 0 & 1/\sqrt{5} \end{bmatrix} \begin{bmatrix} \sqrt{5} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & 0 & 0 \end{bmatrix}^T$$

be an SVD.

   (a) What is the rank of A?

   (b) Find bases of the four fundamental subspaces of A.

   (c) What are the eigenvalues of $A^T A$?

   (d) Are the columns of A are linearly independent?

2. Prove that $N(A^TA) = N(A)$ as follows.

   (a) Show $N(A) \subseteq N(A^TA)$.

   (b) Show $N(A^TA) \subseteq N(A)$ (hint: notice that $\mathbf{x}^T(A^TA)\mathbf{x} = 0$ for all $\mathbf{x} \in N(A^TA)$)

   (c) Deduce that $N(A^TA) = N(A)$.

3. Open the **Julia13SVD** notebook to learn about the SVD command and complete the problems in the notebook.